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# Sobolev versus Hölder local minimizers and existence of multiple solutions for a singular quasilinear equation

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**Abstract.** We investigate the following quasilinear and singular problem,

$$\begin{cases} -\Delta_p u = \dfrac{\lambda}{u^\delta} + u^q & \text{in } \Omega; \\ u|_{\partial\Omega} = 0, \quad u > 0 & \text{in } \Omega, \end{cases} \tag{P}$$

where $\Omega$ is an open bounded domain with smooth boundary, $1 < p < \infty$, $p-1 < q \le p^* - 1$, $\lambda > 0$, and $0 < \delta < 1$. As usual, $p^* = \frac{Np}{N-p}$ if $1 < p < N$, $p^* \in (p, \infty)$ is arbitrarily large if $p = N$, and $p^* = \infty$ if $p > N$. We employ variational methods in order to show the existence of at least two distinct (positive) solutions of problem (P) in $W_0^{1,p}(\Omega)$. While following an approach due to Ambrosetti-Brezis-Cerami, we need to prove two new results of separate interest: a strong comparison principle and a regularity result for solutions to problem (P) in $C^{1,\beta}(\overline{\Omega})$ with some $\beta \in (0, 1)$. Furthermore, we show that $\delta < 1$ is a reasonable sufficient (and likely optimal) condition to obtain solutions of problem (P) in $C^1(\overline{\Omega})$.

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