Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. V (2006), 483-548

# The $BV$-energy of maps into a manifold: relaxation and density results

MARIANO GIAQUINTA AND DOMENICO MUCCI

**Abstract.** Let $\mathcal{Y}$ be a smooth compact oriented Riemannian manifold without boundary, and assume that its 1-homology group has no torsion. Weak limits of graphs of smooth maps $u_k : B^n \to \mathcal{Y}$ with equibounded total variation give rise to equivalence classes of Cartesian currents in $\mathrm{cart}^{1,1}(B^n \times \mathcal{Y})$ for which we introduce a natural $BV$-energy. Assume moreover that the first homotopy group of $\mathcal{Y}$ is commutative. In any dimension $n$ we prove that every element $T$ in $\mathrm{cart}^{1,1}(B^n \times \mathcal{Y})$ can be approximated weakly in the sense of currents by a sequence of graphs of smooth maps $u_k : B^n \to \mathcal{Y}$ with total variation converging to the $BV$-energy of $T$. As a consequence, we characterize the lower semicontinuous envelope of functions of bounded variations from $B^n$ into $\mathcal{Y}$.

**Mathematics Subject Classification (2000):** 49Q15 (primary); 49Q20 (secondary).