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One-dimensional symmetry of periodic minimizers for a mean field equation

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Abstract. We consider on a two-dimensional flat torus T defined by a rectangular periodic cell the following equation

$$\Delta u + \rho \left(\frac{e^u}{\int_T e^u} - \frac{1}{|T|} \right) = 0, \quad \int_T u = 0.$$

It is well-known that the associated energy functional admits a minimizer for each $\rho \leq 8\pi$. The present paper shows that these minimizers depend actually only on one variable. As a consequence, setting $\lambda_1(T)$ to be the first eigenvalue of the Laplacian on the torus, the minimizers are identically zero whenever $\rho \leq \min\{8\pi, \lambda_1(T)|T|\}$. Our results hold more generally for solutions that are Steiner symmetric, up to a translation.

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