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Persistence of Coron's solution in nearly critical problems

MONICA MUSSO AND ANGELA PISTOIA

Abstract. We consider the problem

$$\begin{cases} -\Delta u = u^{\frac{N+2}{N-2}+\lambda} & \text{in } \Omega \setminus \varepsilon\omega, \\ u > 0 & \text{in } \Omega \setminus \varepsilon\omega, \\ u = 0 & \text{on } \partial\left(\Omega \setminus \varepsilon\omega\right), \end{cases}$$

where Ω and ω are smooth bounded domains in $\mathbb{R}^N$, $N \geq 3$, $\varepsilon > 0$ and $\lambda \in \mathbb{R}$. We prove that if the size of the hole ε goes to zero and if, simultaneously, the parameter λ goes to zero at the appropriate rate, then the problem has a solution which blows up at the origin.

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