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Convex integration and the L^p theory of elliptic equations

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Abstract. This paper deals with the L^p theory of linear elliptic partial differential equations with bounded measurable coefficients. We construct in two dimensions examples of weak and so-called very weak solutions, with critical integrability properties, both to isotropic equations and to equations in non-divergence form. These examples show that the general L^p theory, developed in [1, 24] and [2], cannot be extended under any restriction on the essential range of the coefficients. Our constructions are based on the method of convex integration, as used by S. Müller and V. Šverák in [30] for the construction of counterexamples to regularity in elliptic systems, combined with the staircase type laminates introduced in [15].

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