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The role of Onofri type inequalities in the symmetry properties of extremals for Caffarelli-Kohn-Nirenberg inequalities, in two space dimensions

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Abstract. We first discuss a class of inequalities of Onofri type depending on a parameter, in the two-dimensional Euclidean space. The inequality holds for radial functions if the parameter is larger than -1. Without symmetry assumption, it holds if and only if the parameter is in the interval $(-1, 0]$.

The inequality gives us some insight on the symmetry breaking phenomenon for the extremal functions of the Caffarelli-Kohn-Nirenberg inequality, in two space dimensions. In fact, for suitable sets of parameters (asymptotically sharp) we prove symmetry or symmetry breaking by means of a blow-up method and a careful analysis of the convergence to a solution of a Liouville equation. In this way, the Onofri inequality appears as a limit case of the Caffarelli-Kohn-Nirenberg inequality.

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