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Bernstein and De Giorgi type problems: new results via a geometric approach

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Abstract. We use a Poincaré type formula and level set analysis to detect one-dimensional symmetry of stable solutions of possibly degenerate or singular elliptic equations of the form

$$\operatorname{div}\Big(a(|\nabla u(x)|)\nabla u(x)\Big) + f(u(x)) = 0.$$

Our setting is very general and, as particular cases, we obtain new proofs of a conjecture of De Giorgi for phase transitions in $\mathbb{R}^2$ and $\mathbb{R}^3$ and of the Bernstein problem on the flatness of minimal area graphs in $\mathbb{R}^3$. A one-dimensional symmetry result in the half-space is also obtained as a byproduct of our analysis. Our approach is also flexible to very degenerate operators: as an application, we prove one-dimensional symmetry for 1-Laplacian type operators.

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