Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. VIII (2009), 613-646

Derivative loss for Kirchhoff equations with non-Lipschitz nonlinear term

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Abstract. In this paper we consider the Cauchy boundary value problem for the integro-differential equation

$$u_{tt} - m\left(\int_\Omega |\nabla u|^2\, dx\right)\Delta u = 0 \qquad \text{in } \Omega \times [0, T)$$

with a continuous nonlinearity $m : [0, +\infty) \to [0, +\infty)$.

It is well known that a *local* solution exists provided that the initial data are regular enough. The required regularity depends on the continuity modulus of m.

In this paper we present some counterexamples in order to show that the regularity required in the existence results is sharp, at least if we want solutions with the same space regularity of initial data. In these examples we construct indeed local solutions which are regular at $t = 0$, but exhibit an instantaneous (often infinite) derivative loss in the space variables.

Mathematics Subject Classification (2000): 35L70 (primary); 35L80, 35L90 (secondary).