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Existence of weak solutions for unsteady motions of generalized Newtonian fluids

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Abstract. We prove the existence of weak solutions $\mathbf{u} : Q_T \to \mathbb{R}^n$ of the equations of unsteady motion of an incompressible fluid with shear-dependent viscosity in a cylinder $Q_T = \Omega \times (0, T)$, where $\Omega \subset \mathbb{R}^n$ denotes a bounded domain. Under the assumption that the extra stress tensor **S** possesses a *q*-structure with $q > \frac{2n}{n+2}$, we are able to construct a weak solution $\mathbf{u} \in L^q(0, T; W_0^{1,q}(\Omega)) \cap C_w([0, T]; L^2(\Omega))$ with div $\mathbf{u} = 0$. Our approach is based on the Lipschitz truncation method, which is new in this context.

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