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On square roots of class C^m of nonnegative functions of one variable

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Abstract. We investigate the regularity of functions g such that $g^2 = f$, where f is a given nonnegative function of one variable. Assuming that f is of class C^{2m} ($m > 1$) and vanishes together with its derivatives up to order $2m - 4$ at all its local minimum points, one can find a g of class C^m. Under the same assumption on the minimum points, if f is of class C^{2m+2} then g can be chosen such that it admits a derivative of order $m + 1$ everywhere. Counterexamples show that these results are sharp.

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