Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. IX (2010), 785-813

Integral pinching results for manifolds with boundary

GIOVANNI CATINO AND CHEIKH BIRAHIM NDIAYE

Abstract. We prove that some Riemannian manifolds with boundary satisfying an explicit integral pinching condition are spherical space-forms. More precisely, we show that three-dimensional Riemannian manifolds with totally geodesic boundary, positive scalar curvature and an explicit integral pinching between the L^2-norm of the scalar curvature and the L^2-norm of the Ricci tensor are spherical space-forms with totally geodesic boundary. Moreover, we also prove that four-dimensional Riemannian manifolds with umbilic boundary, positive Yamabe invariant and an explicit integral pinching between the total integral of the (Q,T)-curvature and the L^2-norm of the Weyl curvature are spherical space-forms with totally geodesic boundary. As a consequence, we show that a certain conformally invariant operator, which plays an important role in Conformal Geometry, is non-negative and has trivial kernel if the Yamabe invariant is positive and verifies a pinching condition together with the total integral of the (Q,T)-curvature. As an application of the latter spectral analysis, we show the existence of conformal metrics with constant Q-curvature, constant T-curvature, and zero mean curvature under the latter assumptions.

Mathematics Subject Classification (2010): 53C24 (primary); 53C20, 53C21, 53C25 (secondary).