Regularity for the CR vector bundle problem II

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Abstract. We derive a $\mathcal{C}^{k+\frac{1}{2}}$ Hölder estimate for $P\varphi$, where P is either of the two solution operators in Henkin's local homotopy formula for $\overline{\partial}_b$ on a strongly pseudoconvex real hypersurface M in $\mathbb{C}^n$, φ is a $(0,q)$-form of class $\mathcal{C}^k$ on M, and $k \geq 0$ is an integer. We also derive a $\mathcal{C}^a$ estimate for $P\varphi$, when φ is of class $\mathcal{C}^a$ and $a \geq 0$ is a real number. These estimates require that M be of class $\mathcal{C}^{k+\frac{5}{2}}$, or $\mathcal{C}^{a+2}$, respectively. The explicit bounds for the constants occurring in these estimates also considerably improve previously known such results.

These estimates are then applied to the integrability problem for CR vector bundles to gain improved regularity. They also constitute a major ingredient in a forthcoming work of the authors on the local CR embedding problem.

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