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A variational method for a class of parabolic PDEs

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Abstract. In this manuscript we extend De Giorgi's interpolation method to a class of parabolic equations which are not gradient flows but possess an entropy functional and an underlying Lagrangian. The new fact in the study is that not only the Lagrangian may depend on spatial variables, but it does not induce a metric. Assuming the initial condition to be a density function, not necessarily smooth, but solely of bounded first moments and finite "entropy", we use a variational scheme to discretize the equation in time and construct approximate solutions. Then De Giorgi's interpolation method is revealed to be a powerful tool for proving convergence of our algorithm. Finally we show uniqueness and stability in L^1 of our solutions.

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