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Nonlinear potentials, local solutions to elliptic equations and rearrangements

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Abstract. A sharp rearrangement estimate for the nonlinear Havin-Maz'ya potentials is established. In particular, this estimate leads to a characterization of those rearrangement invariant spaces between which the nonlinear potentials are bounded. In combination with results from [24] and [18], it also enables us to derive local bounds for solutions to quasilinear elliptic PDE's and for their gradient in rearrangement form. As a consequence, the local regularity of solutions to elliptic equations and for their gradient in arbitrary rearrangement invariant spaces is reduced to one-dimensional Hardy-type inequalities. Applications to the special cases of Lorentz and Orlicz spaces are presented.

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