Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5) Vol. X (2011), 913-984

Boundary trace of positive solutions of semilinear elliptic equations in Lipschitz domains: the subcritical case

MOSHE MARCUS AND LAURENT VERON

Abstract. We study the generalized boundary value problem for nonnegative solutions of $-\Delta u + g(u) = 0$ in a bounded Lipschitz domain Ω , when g is continuous and nondecreasing. Using the harmonic measure of Ω , we define a trace in the class of outer regular Borel measures. We amphasize the case where $g(u) = |u|^{q-1}u, q > 1$. When Ω is (locally) a cone with vertex y, we prove sharp results of removability and characterization of singular behavior. In the general case, assuming that Ω possesses a tangent cone at every boundary point and q is subcritical, we prove an existence and uniqueness result for positive solutions with arbitrary boundary trace.

Mathematics Subject Classification (2010): 35K60 (primary); 31A20, 31C15, 44A25, 46E35 (secodary).