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On the entangled ergodic theorem

TANJA EISNER AND DÁVID KUNSZENTI-KOVÁCS

Abstract. We study the convergence of the so-called entangled ergodic averages

$$\frac{1}{N^k} \sum_{n_1,\dots,n_k=1}^{N} T_m^{n_{\alpha(m)}} A_{m-1} T_{m-1}^{n_{\alpha(m-1)}} A_{m-2} \dots A_1 T_1^{n_{\alpha(1)}},$$

where $k \leq m$ and $\alpha : \{1, \dots, m\} \to \{1, \dots, k\}$ is a surjective map. We show that, on general Banach spaces and without any restriction on the partition α, the above averages converge strongly as $N \to \infty$ under some quite weak compactness assumptions on the operators T_j and A_j. A formula for the limit based on the spectral analysis of the operators T_j and the continuous version of the result are presented as well.

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