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Multiply monogenic orders

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Abstract. Let $A = \mathbb{Z}[x_1, \dots, x_r] \supset \mathbb{Z}$ be a domain which is finitely generated over $\mathbb{Z}$ and integrally closed in its quotient field L. Further, let K be a finite extension field of L. An A-order in K is a domain $\mathcal{O} \supset A$ with quotient field K which is integral over A. A-orders in K of the type $A[\alpha]$ are called monogenic. It was proved by Győry [10] that for any given A-order $\mathcal{O}$ in K there are at most finitely many A-equivalence classes of $\alpha \in \mathcal{O}$ with $A[\alpha] = \mathcal{O}$, where two elements α, β of $\mathcal{O}$ are called A-equivalent if $\beta = u\alpha + a$ for some $u \in A^*$, $a \in A$. If the number of A-equivalence classes of α with $A[\alpha] = \mathcal{O}$ is at least k, we call $\mathcal{O}$ k times monogenic.

In this paper we study orders which are more than one time monogenic. Our first main result is that if K is any finite extension of L of degree ≥ 3, then there are only finitely many three times monogenic A-orders in K. Next, we define two special types of two times monogenic A-orders, and show that there are extensions K which have infinitely many orders of these types. Then under certain conditions imposed on the Galois group of the normal closure of K over L, we prove that K has only finitely many two times monogenic A-orders which are not of these types. Some immediate applications to canonical number systems are also mentioned.

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