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A priori estimates and existence for elliptic equations with gradient dependent terms

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Abstract. We consider, in a bounded domain $\Omega \subset \mathbb{R}^N$, a class of nonlinear elliptic equations in divergence form as

$$\begin{cases} \alpha_0\, u - \operatorname{div}(a(x, u, Du)) = H(x, u, Du) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $\alpha_0 \geq 0$, the second order part is a coercive, pseudomonotone operator of Leray-Lions type in the Sobolev space $W_0^{1,p}(\Omega)$, $p > 1$, and the function H grows at most like $|Du|^q + f(x)$, with $p - 1 < q < p$. Assuming $f(x)$ to belong to an (optimal) Lebesgue class L^m, with $m < \frac{N}{p}$, we prove a priori estimates and existence of solutions, discussing several ranges of the exponents m, q and p which include cases of singular data (L^1 data or measures). The obtention of a priori estimates is not straightforward because of the "superlinear"character of the first order terms. To this purpose we use a new approach, generalizing the method introduced in our note [29]. We complete the results known in the previous literature where either $q \leq p - 1$ or $m \geq \frac{N}{p}$.

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