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Deformations of constant mean curvature surfaces preserving symmetries and the Hopf differential

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Abstract. We define certain deformations between minimal and non-minimal constant mean curvature (CMC) surfaces in Euclidean space $\mathbb{E}^3$ which preserve the Hopf differential. We prove that, given a CMC H surface f, either minimal or not, and a fixed basepoint z_0 on this surface, there is a naturally defined family f_h, for all $h \in \mathbb{R}$, of CMC h surfaces that are tangent to f at z_0, and which have the same Hopf differential. Given the classical Weierstrass data for a minimal surface, we give an explicit formula for the generalized Weierstrass data for the non-minimal surfaces f_h, and vice versa. As an application, we use this to give a well-defined dressing action on the class of minimal surfaces. In addition, we show that symmetries of certain types associated with the basepoint are preserved under the deformation, and this gives a canonical choice of basepoint for surfaces with symmetries. We use this to define new examples of non-minimal CMC surfaces naturally associated to known minimal surfaces with symmetries.

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