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Motion by Curvature of Planar Networks

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Abstract. We consider the motion by curvature of a network of smooth curves with multiple junctions in the plane, that is, the geometric gradient flow associated to the length functional.
Such a flow represents the evolution of a two–dimensional multiphase system where the energy is simply the sum of the lengths of the interfaces, in particular it is a possible model for the growth of grain boundaries.
Moreover, the motion of these networks of curves is the simplest example of curvature flow for sets which are "essentially" non regular.
As a first step, in this paper we study in detail the case of three curves in the plane meeting at a single triple junction and with the other ends fixed. We show some results about the existence, uniqueness and, in particular, the global regularity of the flow, following the line of analysis carried on in the last years for the evolution by mean curvature of smooth curves and hypersurfaces.

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