Ricci surfaces

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Abstract. A Ricci surface is a Riemannian 2-manifold (M, g) whose Gaussian curvature K satisfies $K \Delta K + g(dK, dK) + 4K^3 = 0$. Every minimal surface isometrically embedded in $\mathbb{R}^3$ is a Ricci surface of non-positive curvature. At the end of the 19$^{\text{th}}$ century Ricci-Curbastro has proved that, conversely, every point x of a Ricci surface has a neighborhood which embeds isometrically in $\mathbb{R}^3$ as a minimal surface, provided $K(x) < 0$. We prove this result in full generality by showing that Ricci surfaces can be locally isometrically embedded either minimally in $\mathbb{R}^3$ or maximally in $\mathbb{R}^{2,1}$, including near points of vanishing curvature. We then develop the theory of closed Ricci surfaces, possibly with conical singularities, and construct classes of examples in all genera $g \geq 2$.