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On the Second Order Derivatives of Convex Functions on the Heisenberg Group

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Abstract. In the Euclidean setting the celebrated Aleksandrov-Busemann-Feller theorem states that convex functions are a.e. twice differentiable. In this paper we prove that a similar result holds in the Heisenberg group, by showing that every continuous \mathcal{H} -convex function belongs to the class of functions whose second order horizontal distributional derivatives are Radon measures. Together with a recent result by Ambrosio and Magnani, this proves the existence a.e. of second order horizontal derivatives for the class of continuous \mathcal{H} -convex functions in the Heisenberg group.

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