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Allen-Cahn approximation of mean curvature flow in Riemannian manifolds I, uniform estimates

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Abstract. We are concerned with solutions to the parabolic Allen-Cahn equation in Riemannian manifolds. For a general class of initial conditions we show non-positivity of the limiting energy discrepancy. This in turn allows us to prove an almost monotonicity formula (a weak counterpart of Huisken's monotonicity formula) which gives a local uniform control of the energy densities at small scales.

These results will be used in [41] to extend previous important results from [31] in Euclidean space, showing convergence of solutions to the parabolic Allen-Cahn equations to Brakke's motion by mean curvature in Riemannian manifolds.

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