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Sharp Estimates for the Ornstein-Uhlenbeck Operator

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Abstract. Let $\mathcal{L}$ be the Ornstein-Uhlenbeck operator which is self-adjoint with respect to the Gauss measure γ on $\mathbb{R}^d$. We prove a sharp estimate of the operator norm of the imaginary powers of $\mathcal{L}$ on $L^p(\gamma)$, $1 < p < \infty$. Then we use this estimate to prove that if b is in $[0, \infty)$ and M is a bounded holomorphic function in the sector $\{z \in \mathbb{C} : |\arg(z - b)| < \arcsin|2/p - 1|\}$ and satisfies a Hörmander-like condition of (nonintegral) order greater than one on the boundary, then the operator $M(\mathcal{L})$ is bounded on $L^p(\gamma)$. This improves earlier results of the authors with J. García-Cuerva and J.L. Torrea.

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