Criterion of the L^2 boundedness and sharp endpoint estimates for singular integral operators on product spaces of homogeneous type

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Abstract. The purpose of this paper is to introduce a class of general singular integral operators on spaces $\widetilde{M} = M_1 \times \cdots \times M_n$. Each factor space M_i, $1 \leq i \leq n$, is a space of homogeneous type in the sense of Coifman and Weiss. These operators generalize those studied by Journé on the Euclidean space and include operators studied by Nagel and Stein on Carnot-Carathéodory spaces on which the basic geometry is given by a control, or Carnot-Carathéodory, metric induced by a collection of vector fields of finite type. We provide the criterion of the $L^2(\widetilde{M})$ boundedness for these general operators. Thus this result extends the product $T1$ theorem of Journé on Euclidean space and recovers the L^p, $1 < p < \infty$, boundedness of those operators on Carnot-Carathéodory space obtained by Nagel and Stein. We also prove the sharp endpoint estimates for these general operators on the Hardy spaces $H^p(\widetilde{M})$ and $BMO(\widetilde{M})$.

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