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Construction of a stable blow-up solution for a class of strongly perturbed semilinear heat equations

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Abstract. We construct a solution for a class of perturbed semilinear heat equations which blows up in finite time with a prescribed blow-up profile. The construction relies on the reduction of the problem to a finite-dimensional one, and on the use of index theory for the conclusion. When the perturbation is in some sense weak, say polynomial, the construction initiated by Bricmont and Kupiainen [5], then parsued by Merle and Zaag [25], works with very minor adaptations. However, when the perturbation is stronger, say in logarithmic scales with respect to the main nonlinear term, a direct application of the methods of [5] and [25] is not successful. Truly new ideas are needed to perform the construction, in which the substantial novelty of our paper resides. As in earlier works, a geometric interpretation of the parameters of the finite-dimensional problem yields the stability of the constructed solution.

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