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Almost nef regular foliations and Fujita's decomposition of reflexive sheaves

MASATAKA IWAI

Abstract. In this paper we study almost nef regular foliations. We give a structure theorem of a smooth projective variety X with an almost nef regular foliation $\mathcal{F}$: X admits a smooth morphism $f : X \to Y$ with rationally connected fibers such that $\mathcal{F}$ is a pullback of a numerically flat regular foliation on Y. Moreover, f is characterized as a relative MRC fibration of an algebraic part of $\mathcal{F}$. As a corollary, an almost nef tangent bundle of a rationally connected variety is generically ample. For the proof, we generalize Fujita's decomposition theorem. As a by-product, we show that a reflexive hull of $f_*(mK_{X/Y})$ is a direct sum of a Hermitian flat vector bundle and a generically ample reflexive sheaf for any algebraic fiber space $f : X \to Y$. We also study foliations with nef anti-canonical bundles.

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On the other hand, even if a tangent bundle T_X of a smooth projective variety X contains a subsheaf $\mathcal{F}$ with some algebraic positivity, then the structure of X is expected to be restricted. In [2] and [35], if T_X contains a rank r ample subsheaf $\mathcal{F}$, then X is isomorphic to $\mathbb{CP}^n$ and $\mathcal{F}$ is isomorphic to either $T_{\mathbb{CP}^n}$ or $\mathcal{O}_{\mathbb{CP}^n}(1)^{\oplus r}$.

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