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A topological invariant of line arrangements

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Abstract. We define a new topological invariant of line arrangements in the complex projective plane. This invariant is a root of unity defined under some combinatorial restrictions for arrangements endowed with some special torsion character on the fundamental group of their complement. It is derived from the peripheral structure on the group induced by the inclusion map of the boundary of a tubular neigborhood in the exterior of the arrangement. By similarity with knot theory, it can be viewed as an analogue of linking numbers. This is an orientation-preserving invariant for ordered arrangements. We give an explicit method to compute the invariant from the equations of the arrangement, using the wiring diagrams introduced by Arvola, that encode the braid monodromy. Moreover, this invariant is a crucial ingredient to compute the depth of a character satisfying some resonant conditions, and completes the existent methods by Libgober and the first author. Finally, we compute the invariant for extended MacLane arrangements with an additional line and observe that it takes different values for the deformation classes.

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