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Polynomial semiconjugacies, decompositions of iterations, and invariant curves

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Abstract. We study the functional equation $A \circ X = X \circ B$, where A, B, and X are polynomials with complex coefficients. Using results of [13] about polynomials sharing preimages of compact sets in $\mathbb{C}$, we show that for given B its solutions may be described in terms of the filled-in Julia set of B. On this base, we prove a number of results describing a general structure of solutions. The results obtained imply in particular the result of Medvedev and Scanlon [10] about invariant curves of maps $F : \mathbb{C}^2 \to \mathbb{C}^2$ of the form $(x, y) \to (f(x), f(y))$, where f is a polynomial, and a version of the result of Zieve and Müller [22] about decompositions of iterations of a polynomial.

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