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Quartic differentials and harmonic maps in conformal surface geometry

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Abstract. We consider codimension 2 sphere congruences in pseudo-conformal geometry that are harmonic with respect to the conformal structure of an orthogonal surface. We characterise the orthogonal surfaces of such congruences as either S-Willmore surfaces, quasi-umbilical surfaces, constant mean curvature surfaces in 3-dimensional space forms or surfaces of constant lightcone mean curvature in 3-dimensional lightcones. We then investigate Bryant's quartic differential in this context and show that generically this is divergence free if and only if the surface under consideration is either superconformal or orthogonal to a harmonic congruence of codimension 2 spheres. We may then apply the previous result to characterise surfaces with such a property.

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