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Global well-posedness for the non-linear Maxwell-Schrödinger system

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Abstract. In this paper we study the Cauchy problem associated to the Maxwell-Schrödinger system with a defocusing pure-power non-linearity. This system has many applications in physics, for instance in the description of a charged non-relativistic quantum plasma, interacting with its self-generated electromagnetic potential. One consequence of our analysis is to demonstrate that the Lorentz force associated with the electromagnetic field is well-defined for solutions slightly more regular than the finite energy class. This aspect is of fundamental importance since all the related physical models require the observability of electromagnetic effects. The well-posedness of the Lorentz force still seems to be a major open problem in the class of solutions which are only finite energy. We show the global well-posedness at high regularity for the cubic and sub-cubic case, and we provide polynomial bounds for the growth of the Sobolev norm of the solutions, for a certain range of non-linearities. An important role is played by appropriate a priori dispersive estimates, obtained by means of Koch-Tzvetkov type bounds for the non-homogeneous Schrödinger equation, which overcome the lack of Strichartz estimates for the magnetic Schrödinger flow. Because of the power-type non-linearity, the propagation of higher regularity, globally in time, cannot be achieved via a bootstrap argument as done in [45]. Our approach then exploits the analysis of a modified energy functional, combined with the a priori bounds coming from the dispersive estimates obtained previously.

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