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On Volumes of Arithmetic Quotients of $SO(1,n)$

MIKHAIL BELOLIPETSKY

Abstract. We apply G. Prasad's volume formula for the arithmetic quotients of semi-simple groups and Bruhat-Tits theory to study the covolumes of arithmetic subgroups of $SO(1,n)$. As a result we prove that for any even dimension n there exists a unique compact arithmetic hyperbolic n-orbifold of the smallest volume. We give a formula for the Euler-Poincaré characteristic of the orbifolds and present an explicit description of their fundamental groups as the stabilizers of certain lattices in quadratic spaces. We also study hyperbolic 4-manifolds defined arithmetically and obtain a number theoretical characterization of the smallest compact arithmetic 4-manifold.

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