Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXIII (2022), 1597-1624

Pointwise universal Gysin formulæ and applications towards Griffiths' conjecture

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Abstract. Let X be a complex manifold, $(E,h) \to X$ be a rank r holomorphic Hermitian vector bundle, and ρ be a sequence of dimensions $0 = \rho_0 < \rho_1 < \cdots < \rho_m = r$. Let $Q_{\rho,j}$, $j = 1,\dots,m$, be the tautological line bundles over the (possibly incomplete) flag bundle $\mathbb{F}_\rho(E) \to X$ associated to ρ, endowed with the natural metrics induced by that of E, with Chern curvatures $\Xi_{\rho,j}$. We show that the universal Gysin formula *à la* Darondeau-Pragacz for the push-forward of a homogeneous polynomial in the Chern classes of the $Q_{\rho,j}$'s also holds pointwise at the level of the Chern forms $\Xi_{\rho,j}$ in this Hermitianized situation.

As an application, we show the strong positivity of several polynomials in the Chern forms of a Griffiths (semi)positive vector bundle not previously known, thus giving some new evidence towards a conjecture by Griffiths, which in turn can be seen as a pointwise Hermitianized version of the Fulton-Lazarsfeld theorem on numerically positive polynomials for ample vector bundles.

Mathematics Subject Classification (2020): 32L05 (primary); 14M15, 57R20 (secondary).