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The diophantine problem for rings of exponential polynomials

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Abstract. One of the main open problems regarding decidability of the existential theory of rings is the analogue of Hilbert's Tenth Problem (HTP) for the ring of entire holomorphic functions in one variable. In the direction of a negative solution, we prove unsolvability of HTP for rings of exponential polynomials. This provides the first known case of HTP for a ring of entire holomorphic functions in one variable strictly containing the polynomials. The technique of proof consists of an interaction between Arithmetic, Analysis, Logic, and Functional Transcendence.

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