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Volume and self-intersection of differences of two nef classes

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Abstract. Let $\{\alpha\}$ and $\{\beta\}$ be nef cohomology classes of bidegree (1, 1) on a compact n-dimensional Kähler manifold X such that the difference of intersection numbers $\{\alpha\}^n - n\,\{\alpha\}^{n-1}.\{\beta\}$ is positive. We solve in a number of special but rather inclusive cases the quantitative part of Demailly's Transcendental Morse Inequalities Conjecture for this context predicting the lower bound $\{\alpha\}^n - n\,\{\alpha\}^{n-1}.\{\beta\}$ for the volume of the difference class $\{\alpha - \beta\}$. We completely solved the qualitative part in an earlier work. We also give general lower bounds for the volume of $\{\alpha - \beta\}$ and show that the self-intersection number $\{\alpha - \beta\}^n$ is always bounded below by $\{\alpha\}^n - n\,\{\alpha\}^{n-1}.\{\beta\}$. We also describe and estimate the relative psef and nef thresholds of $\{\alpha\}$ with respect to $\{\beta\}$ and relate them to the volume of $\{\alpha - \beta\}$. Finally, broadening the scope beyond the Kähler realm, we propose a conjecture relating the balanced and the Gauduchon cones of $\partial\bar\partial$-manifolds which, if proved to hold, would imply the existence of a balanced metric on any $\partial\bar\partial$-manifold.

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