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Existence of cscK metrics on smooth minimal models

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Abstract. Given a compact Kähler manifold X it is interesting to ask whether it admits a constant scalar curvature Kähler (cscK) metric. In this short note we show that there always exist cscK metrics on compact Kähler manifolds with nef canonical bundle, thus on all smooth minimal models, and also on the blowup of any such manifold. This confirms an expectation of Jian-Shi-Song [19] and extends their main result from K_X semi-ample to K_X nef, with a direct proof that does not appeal to the Abundance conjecture. As a byproduct we obtain that the connected component $\mathrm{Aut}_0(X)$ of the automorphism group of a compact Kähler manifold with K_X nef is either trivial or a complex torus.

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