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Sign-changing blowing-up solutions for the Brezis–Nirenberg problem in dimensions four and five

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Abstract. We consider the Brezis-Nirenberg problem

$$-\Delta u = \lambda u + |u|^{p-1}u \qquad \text{in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $N \geq 3$, $p = \frac{N+2}{N-2}$ and $\lambda > 0$.

We prove that, if Ω is symmetric and $N = 4, 5$, there exists a sign-changing solution whose positive part concentrates and blowsup at the center of symmetry of the domain, while the negative part vanishes, as $\lambda \to \lambda_1$, where $\lambda_1 = \lambda_1(\Omega)$ denotes the first eigenvalue of $-\Delta$ on Ω, with zero Dirichlet boundary condition.

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