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Boundary concentrations on segments for the Lin-Ni-Takagi problem

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Abstract. We consider the following singularly perturbed Neumann problem (Lin-Ni-Takagi problem)

$$\varepsilon^2 \Delta u - u + u^p = 0\,, \quad u > 0 \quad \text{in} \quad \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega,$$

where $p > 2$ and Ω is a smooth and bounded domain in $\mathbb{R}^2$. We construct a new class of solutions that consists of a large number of spikes concentrating on a *segment* of the boundary that contains a *strict local minimum* point of the mean curvature function and has the *same* mean curvature at the two end points. We find a continuum limit of ODE systems governing the interactions of spikes and show that the derivative of the mean curvature function acts as *friction force*. Our construction is partly motivated by the construction of CMC surfaces on broken geodesics by Butscher and Mazzeo [10].

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