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Quaternionic Hardy spaces

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Abstract. The theory of slice regular functions of a quaternionic variable extends the notion of holomorphic function to the quaternionic setting. This fast growing theory is already rich of many results and has significant applications. In this setting, the present paper is devoted to introduce and study the quaternionic counterparts of Hardy spaces of holomorphic functions of one complex variable. The basic properties of the theory of quaternionic Hardy spaces are investigated, and in particular a Poisson-type representation formula, the notions of outer function, singular function and inner function are given. A quaternionic (partial) counterpart of the classical H^p-factorization theorem is also proved. This last result assumes a particularly interesting formulation for a large subclass of slice regular functions, where it is obtained in terms of an outer function, a singular function and a quaternionic Blaschke product.

Mathematics Subject Classification (2010): 30G35 (primary); 30H10 (secondary).

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