Equivalence between dimensional contractions in Wasserstein distance and the curvature-dimension condition

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Abstract. The curvature-dimension condition is a generalization of the Bochner inequality to weighted Riemannian manifolds and general metric measure spaces. It is now known to be equivalent to evolution variational inequalities for the heat semigroup, and quadratic Wasserstein distance contraction properties at different times. On the other hand, in a compact Riemannian manifold, it implies a same-time Wasserstein contraction property for this semigroup. In this work we generalize the latter result to metric measure spaces and more importantly prove the converse: contraction inequalities are equivalent to curvature-dimension conditions. Links with functional inequalities are also investigated.