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Harnack inequality for kinetic Fokker-Planck equations with rough coefficients and application to the Landau equation

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Abstract. We extend the De Giorgi-Nash-Moser theory to a class of kinetic Fokker-Planck equations and deduce new results on the Landau-Coulomb equation. More precisely, we first study the Hölder regularity and establish a Harnack inequality for solutions to a general linear equation of Fokker-Planck type whose coefficients are merely measurable and essentially bounded, *i.e.* assuming no regularity on the coefficients in order to later derive results for non-linear problems. This general equation has the formal structure of the hypoelliptic equations "of type II", sometimes also called ultraparabolic equations of Kolmogorov type, but with rough coefficients: it combines a first-order skew-symmetric operator with a second-order elliptic operator involving derivatives along only part of the coordinates and with rough coefficients. These general results are then applied to the non-negative essentially bounded weak solutions of the Landau equation with inverse-power law $\gamma \in [-d, 1]$ whose mass, energy and entropy density are bounded and mass is bounded away from 0, and we deduce the Hölder regularity of these solutions.

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