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On existence and uniqueness for non-autonomous parabolic Cauchy problems with rough coefficients

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Abstract. We consider existence and uniqueness issues for the initial value problem of parabolic equations $\partial_t u = \mathrm{div} A\nabla u$ on the upper half space, with initial data in L^p spaces. The coefficient matrix A is assumed to be uniformly elliptic, but merely bounded measurable in space and time. For real coefficients and a single equation, this is an old topic for which a comprehensive theory is available, culminating in the work of Aronson. Much less is understood for complex coefficients or systems of equations except for the work of Lions, mainly because of the failure of maximum principles. In this paper, we come back to this topic with new methods that do not rely on maximum principles. This allows us to treat systems in this generality when $p \geq 2$, or under certain assumptions such as bounded variation in the time variable (a much weaker assumption that the usual Hölder continuity assumption) when $p < 2$. We reobtain results for real coefficients, and also complement them. For instance, we obtain uniqueness for arbitrary L^p data, $1 \leq p \leq \infty$, in the class $L^\infty(0, T; L^p(\mathbb{R}^n))$. Our approach to the existence problem relies on a careful construction of propagators for an appropriate energy space, encompassing previous constructions. Our approach to the uniqueness problem, the most novel aspect here, relies on a parabolic version of the Kenig-Pipher maximal function, used in the context of elliptic equations on non-smooth domains. We also prove comparison estimates involving conical square functions of Lusin type and prove some Fatou type results about non-tangential convergence of solutions. Recent results on maximal regularity operators in tent spaces that do not require pointwise heat kernel bounds are key tools in this study.

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