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Central limit theorem for probability measures defined by sum-of-digits function in base 2

JORDAN EMME AND PASCAL HUBERT

Abstract. In this paper we prove a central limit theorem for some probability measures defined as asymptotic densities of integer sets defined via sum-of-digit-function. To any non-negative integer a we can associate a measure on $\mathbb{Z}$ called μ_a such that, for any d, $\mu_a(d)$ is the asymptotic density of the set of non-negative integers n such that $s_2(n+a)-s_2(n)=d$ where $s_2(n)$ is the number of digits "1" in the binary expansion of n. We express this probability measure as a product of matrices whose coefficients are operators of $l^1(\mathbb{Z})$. Then we take a sequence of integers $(a_X(n))_{n\in\mathbb{N}}$ defined via a balanced Bernoulli sequence X. We prove that, for almost every sequence, and after renormalization by the typical variance, we have a central limit theorem by computing all the moments and proving that they converge towards the moments of the normal law $\mathcal{N}(0,1)$.

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