Iterated convolutions and endless Riemann surfaces

SHINGO KAMIMOTO AND DAVID SAUZIN

Abstract. We discuss a version of Écalle's definition of resurgence, based on the notion of endless continuability in the Borel plane. We relate this with the notion of Ω -continuability, where Ω is a discrete filtered set or a discrete doubly filtered set, and show how to construct a universal Riemann surface X_{Ω} whose holomorphic functions are in one-to-one correspondence with Ω -continuable functions. We then discuss the Ω -continuability of convolution products and give estimates for iterated convolutions of the form $\hat{\varphi}_1 * \cdots * \hat{\varphi}_n$. This allows us to handle non-linear operations with resurgent series, *e.g.* substitution into a convergent power series.

Mathematics Subject Classification (2010): 34M30 (primary); 40G10, 30B40 (secondary).