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Volume rigidity at ideal points of the character variety of hyperbolic 3-manifolds

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Abstract. Given the fundamental group Γ of a finite-volume complete hyperbolic 3-manifold M, it is possible to associate to any representation $\rho : \Gamma \to \mathrm{Isom}(\mathbb{H}^3)$ a numerical invariant called volume. This invariant is bounded by the hyperbolic volume of M and satisfies a rigidity condition: if the volume of ρ is maximal, then ρ must be conjugated to the holonomy of the hyperbolic structure of M. This paper generalizes this rigidity result by showing that if a sequence of representations of Γ into $\mathrm{Isom}(\mathbb{H}^3)$ satisfies $\lim_{n\to\infty} \mathrm{Vol}(\rho_n) = \mathrm{Vol}(M)$, then there must exist a sequence of elements $g_n \in \mathrm{Isom}(\mathbb{H}^3)$ such that the representations $g_n \circ \rho_n \circ g_n^{-1}$ converge to the holonomy of M. In particular if the sequence $(\rho_n)_{n\in\mathbb{N}}$ converges to an ideal point of the character variety, then the sequence of volumes must stay away from the maximum. In this way we give an answer to [16, Conjecture 1]. We conclude by generalizing the result to the case of k-manifolds and representations in $\mathrm{Isom}(\mathbb{H}^m)$, where $m \geq k \geq 3$.

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