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On anticanonical volumes of weak $\mathbb{Q}$-Fano terminal threefolds of Picard rank two

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Abstract. We show that for a weak $\mathbb{Q}$-Fano threefold X of Picard rank two ($\mathbb{Q}$-factorial with at worst terminal singularities), the anticanonical volume satisfies $-K_X^3 \leq 72$ except in one case, and the equality holds only if $X = \mathbb{P}_{\mathbb{P}^2}(\mathcal{O}_{\mathbb{P}^2} \oplus \mathcal{O}_{\mathbb{P}^2}(3))$. The approach in this article can be served as a general strategy to establish the optimal upper bound of $-K_X^3$ for any canonical Fano threefold, where the described main result serves as the first step.

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