A weak notion of visibility, a family of examples, and Wolff–Denjoy theorems

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Abstract. We investigate a form of visibility introduced recently by Bharali and Zimmer — and shown to be possessed by a class of domains called Goldilocks domains. The range of theorems established for these domains stems from this form of visibility together with certain quantitative estimates that define Goldilocks domains. We show that some of the theorems alluded to follow *merely* from the latter notion of visibility. We call those domains that possess this property visibility domains with respect to the Kobayashi distance. We provide a sufficient condition for a domain in \mathbb{C}^n to be a visibility domain. A part of this paper is devoted to constructing a family of domains that are visibility domains with respect to the Kobayashi distance. Our notion of visibility is reminiscent of uniform visibility in the context of CAT(0) spaces. However, this is an imperfect analogy because, given a bounded domain Ω in \mathbb{C}^n , $n \ge 2$, it is, in general, not even known whether the metric space (Ω, k_{Ω}) (where k_{Ω} is the Kobayashi distance) is a geodesic space. Yet, with just this weak property, we establish two new Wolff–Denjoy-type theorems.

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