Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXII (2021), 691-744

# Discrete versions of the Li-Yau gradient estimate

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**Abstract.** We study positive solutions to the heat equation on graphs. We prove variants of the Li-Yau gradient estimate and the differential Harnack inequality. For some graphs, we can show the estimates to be sharp. We establish new computation rules for differential operators on discrete spaces and introduce a relaxation function that governs the time dependency in the differential Harnack estimate.

**Mathematics Subject Classification (2010):** 35R02 (primary); 35K05, 35K10, 05C10, 05C81 (secondary).