Semi-isometric CR immersions of CR manifolds into Kähler manifolds and applications

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Abstract. We study the second fundamental form of semi-isometric CR immersions from strictly pseudoconvex CR manifolds into Kähler manifolds. As an application, we give a precise condition for the CR umbilicality of real hypersurfaces, extending an well-known theorem by Webster on the nonexistence of CR umbilical points on generic real ellipsoids. As other applications, we extend the linearity theorem of Ji-Yuan for CR immersions into spheres with vanishing second fundamental form to the important case of three-dimensional manifolds, and prove the "first gap" theorem in the spirit of Webster, Faran, Cima-Suffridge, and Huang for semi-isometric CR immersions into a complex Euclidean space of "low" codimension. Our new approach to the linearity theorem is based on the study of the first positive eigenvalue of the Kohn Laplacian.

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