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Galerkin averaging method and Poincaré normal form for some quasilinear PDEs

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Abstract. We use the Galerkin averaging method to construct a coordinate transformation putting a nonlinear PDE in Poincaré normal form up to finite order. We also give a rigorous estimate of the remainder showing that it is small as a differential operator of very high order. The abstract theorem is then applied to a quasilinear wave equation, to the water wave problem and to a nonlinear heat equation. The normal form is then used to construct approximate solutions whose difference from true solutions is estimated. In the case of hyperbolic equations we obtain an estimate of the error valid over time scales of order ϵ^{-1} (ϵ being the norm of the initial datum), as in averaging theorems. For parabolic equations we obtain an estimate of the error valid over infinite time.

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